Boosted Trees

Prof Wells

STA 295: Stat Learning

April 30th, 2024

Outline

- Discuss boosted trees as example of ensemble models
- Implement boosted trees in R

Section 1

Boosting

Motivation

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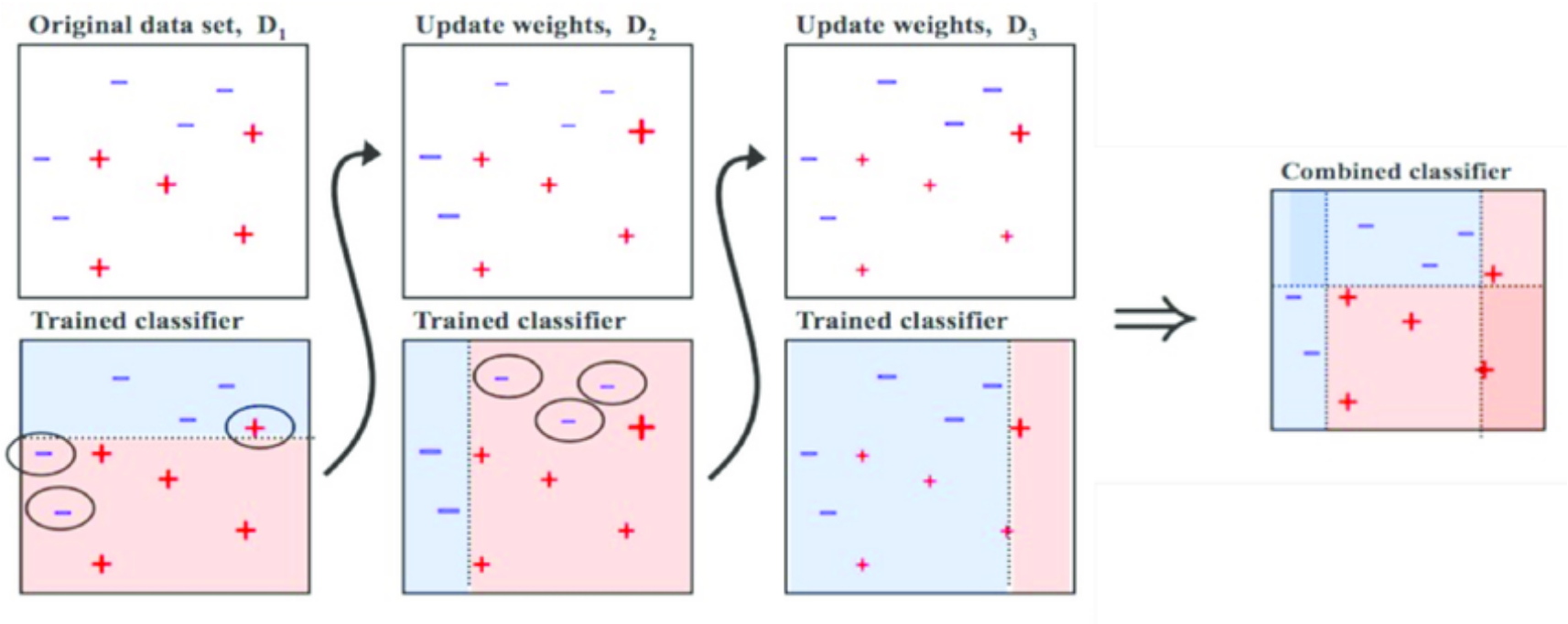
- The overall sequence of classifiers are combined into an ensemble which as high chance of classifying more accurately than any individaul model in the list.
- The algorithm relies on using a sequence of **weak** learners (low variance, high bias)
  - In the tree setting, we can create weak learners by restricting the depth of the tree.

AdaBoost Graphic



Boosting for regression

Boosting also works in the regression setting. The **gradient boosting machine** is a boosting algorithm that works as follows:

1. Select tree depth D and number of iterations K.
2. Compute the average response $\hat{y}$ and use this as the initial predicted value for each observation
3. Compute the residual for each observation.
4. Fit a regression tree of depth D, using the **residuals** as the response.
5. Predict each observation using the regression tree from the previous step.
6. Update the predicted value of each observation by adding the previous iteration's predicted value to the predicted value generated in the previous step.
7. Repeat at total of K times.

Brief Example

We return to the pdxTrees data a final time.

Compute the mean:

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mu <- mean(my_pdxTrees_train$Carbon_Sequestration_lb)
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## [1] 34.49668
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boost_tree_model<- rpart(residuals1 ~ Crown_Base_Height,
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  - Instead of adding the full value for a sample to the previous iteration's predicted value, only a fraction of the current predicted value is added.
  - This fraction is called the *learning rate* λ, with $0 < \lambda < 1$. (Typical values range from 0.001 to 0.01)

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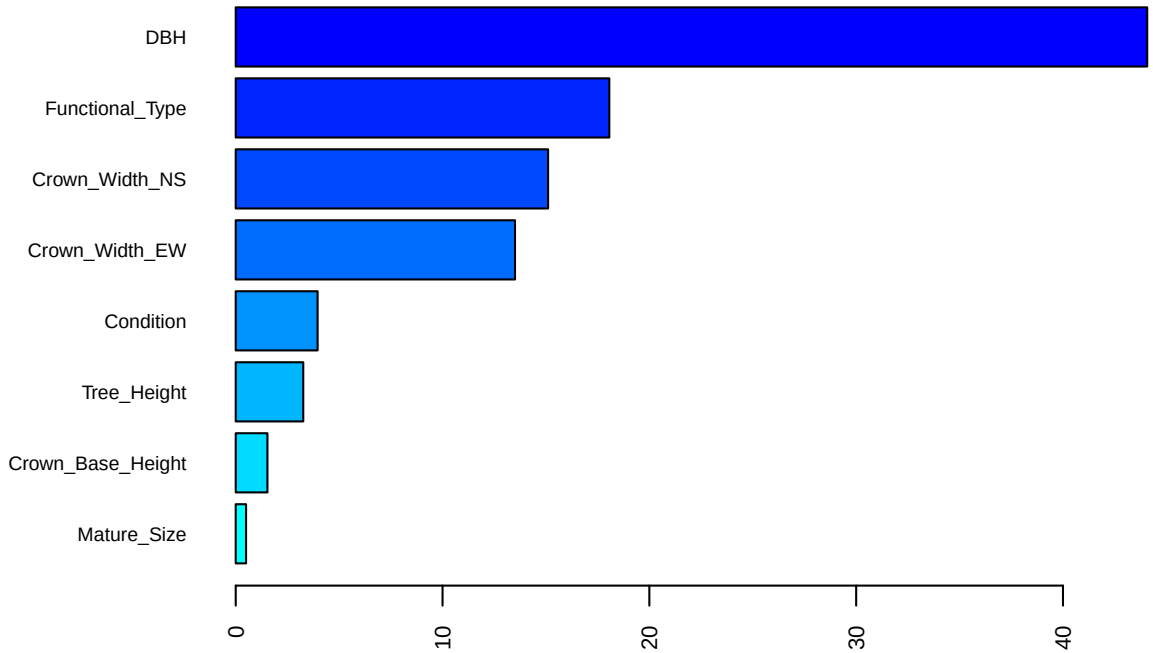
```
library(gbm)
set.seed(10101)
boosted_tree<-gbm(Carbon_Sequestration_lb ~., my_pdxTrees_train,
                  distribution = "gaussian",
                  n.trees=400,
                  interaction.depth = 3,
                  shrinkage = .1)
```

Summary Information

```
summary(boosted_tree )
```

```
##                                 var    rel.inf
## DBH                             DBH 44.0715885
## Functional_Type     Functional_Type 18.0639257
## Crown_Width_NS       Crown_Width_NS 15.1030328
## Crown_Width_EW       Crown_Width_EW 13.5036280
## Condition                 Condition  3.9588168
## Tree_Height             Tree_Height  3.2655545
## Crown_Base_Height Crown_Base_Height  1.5339425
## Mature_Size             Mature_Size  0.4995112
```



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##   model         .metric .estimator .estimate
##   <chr>         <chr>   <chr>          <dbl>
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  - How do we find the best values of these hyperparameters?
  - Cross-validation!

Cross-Validating gbm

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- Cross-validation for gbm models is NOT RECOMMENDED if using the RStudio Server
- We can include an additional cross-validation term in our boosted tree model.
  - It may be helpful to include a number of CPU cores as well. First verify your number of available cores using `parallel::decectCores()`

```
library(gbm)
set.seed(10101)
cv_boosted_tree<-gbm(Carbon_Sequestration_lb ~., my_pdxTrees_train,
                     distribution = "gaussian",
                     n.trees=10000,
                     interaction.depth = 3,
                     shrinkage = .01,
                     cv.folds = 10,
                     n.cores = 8)
```

CV Results

- We can plot cross-validated performance using `gbm.perf()`

```
gbm.perf(cv_boosted_tree, method = "cv")
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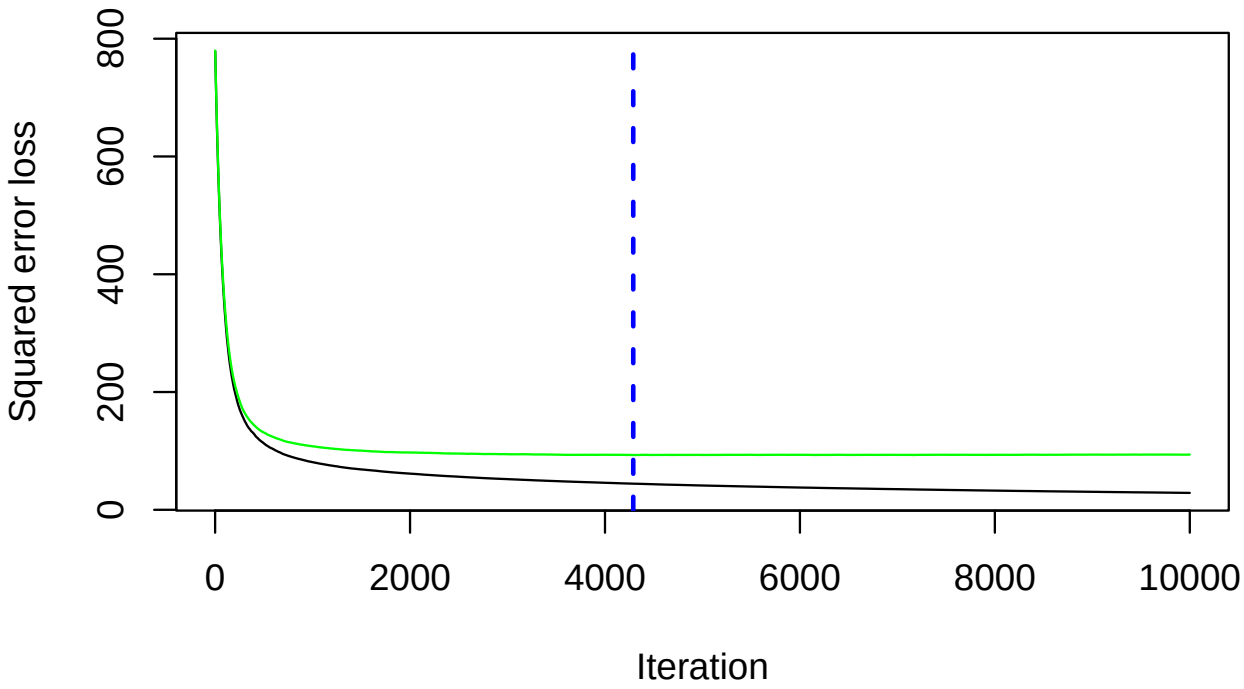


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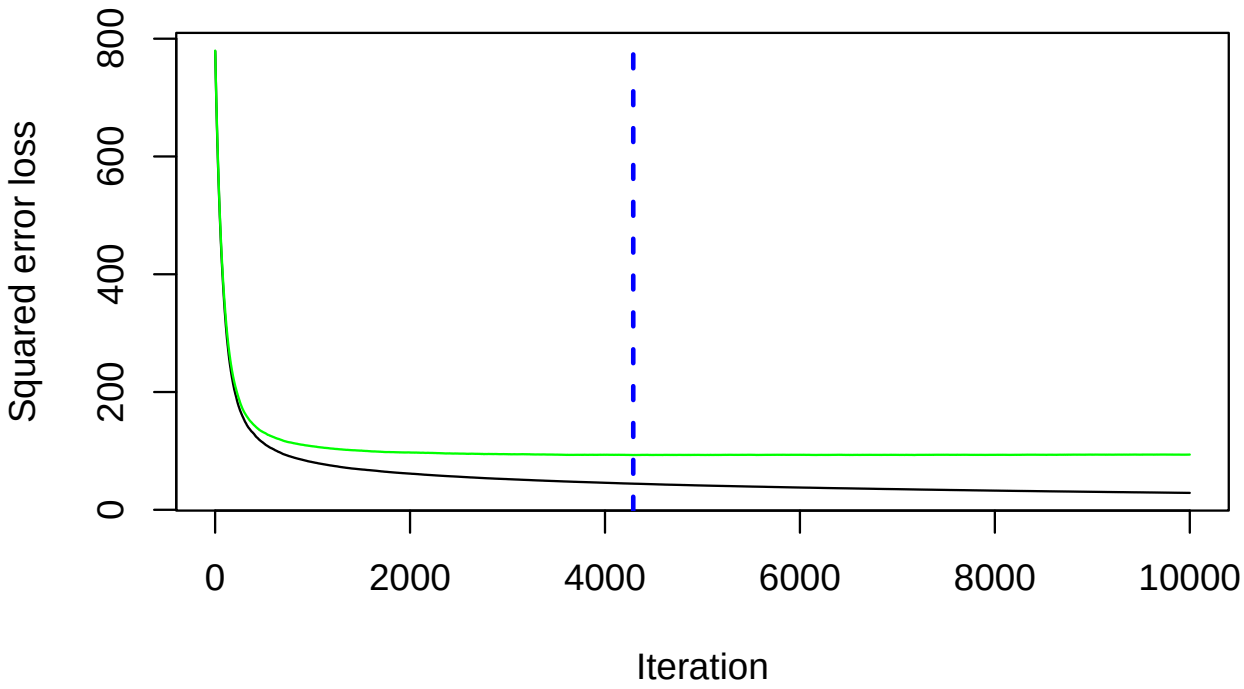


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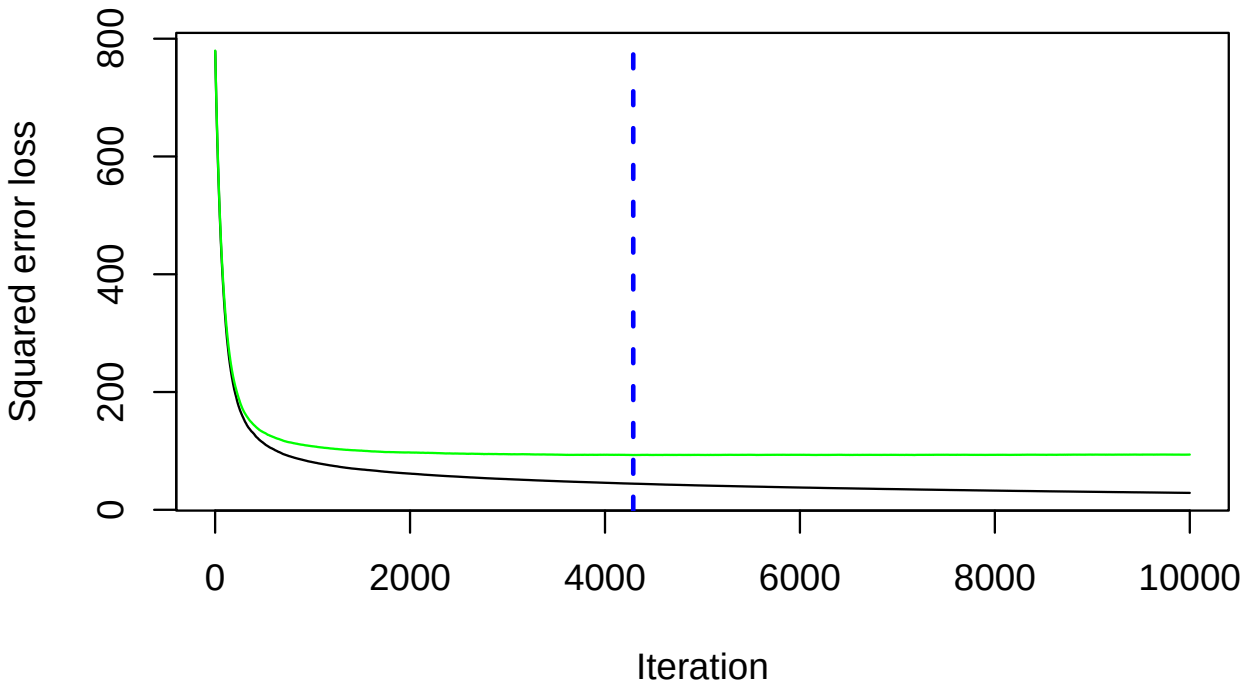


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- The blue vertical line is the optimal value of the cross-validated error

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- This is particularly useful if we want to record the error for a model with certain parameters

General Strategy for finding best Parameters

1. Choose a relatively high initial learning rate. A rate of 0.1 is a reasonable starting point.
2. Determine the optimal number of trees for this learning rate using cross-validation.
3. Fix other tree-specific parameters and tune the learning rate, assessed by computation speed and model accuracy.
4. Tune tree-specific parameters for fixed learning rate.
5. Once tree-specific parameters have been found, lower learning rate and increase number of trees to assess improvements in accuracy.

Warning! This search can take considerable time (minutes to hours), depending on computing power, number of variables in model, and number of observations. DO NOT ATTEMPT ON RSTUDIO SERVER!!