K-Nearest Neighbors

Prof Wells

STA 295: Stat Learning

February 20th, 2024

Outline

In today's class, we will. . .

- Implement KNN in R
- Discuss benefits and drawbacks of KNN

Section 1

K-Nearest Neighbors

KNN Review

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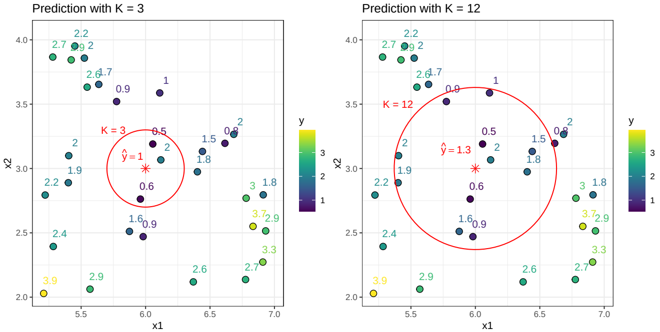

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- In Ch. 5 (next week), we discuss methods for choosing optimal K

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- `kknn` allows us to (optionally) weight observations **by** distance
- `kknn` also allows us to use different notions of distance (Euclidean, Manhattan, Minkowski, Hamming, and more)

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```
##                     Address Latitude Longitude Bedrooms Baths SquareFeet
## 1    1510 First Ave #112  41.73880 -92.71378        2     1       1120
## 2         1020 Center St  41.74558 -92.73168        3     1       1224
## 3      918 Chatterton St  41.74404 -92.71308        4     1       1540
## 4 1023 & 1025 Spring St.  41.74503 -92.72896        3     1       1154
## 5            503 2nd Ave  41.74041 -92.73002        3     1       1277
## 6          9090 Clay St  41.81942 -92.77381        3     1       1079
##     LotSize YearBuilt YearSold MonthSold DaySold OrigPrice ListPrice SalePrice
## 1        NA      1993     2005         9      16     17000     10500      7000
## 2 0.1721763      1900     2006         3      20     35000     35000     27000
## 3        NA      1970     2006         3      15     54000     47000     28000
## 4        NA      1900     2006         2       1     65000     49000     30000
## 5 0.2066116      1900     2005         8      19     35000     35000     30750
## 6 0.1993572      1900     2005         5      27     45900     45900     42000
##      Age
## 1 Modern
## 2    Old
## 3    Mid
## 4    Old
## 5    Old
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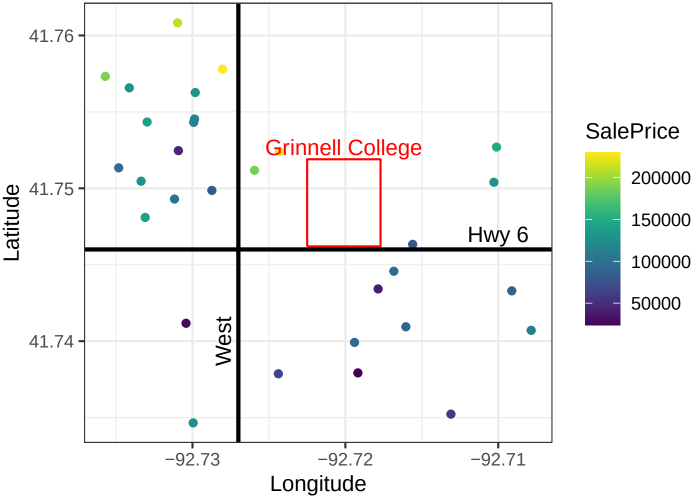
    - The assessor uses the fact that the "value" of the home is exactly what people are willing to pay for similar homes in the same market.
- But this is **exactly** the KNN algorithm.

Location, Location, Location

Let's predict house price, based on Latitude (N-S) and Longitude (E-W)



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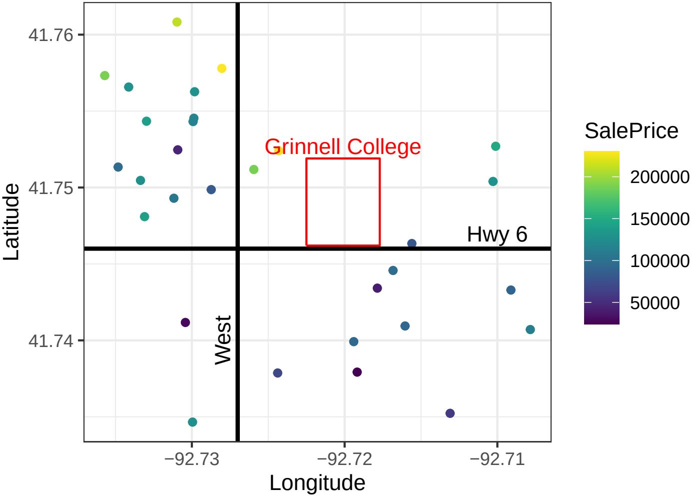
Why is a multilinear model potentially a poor choice?



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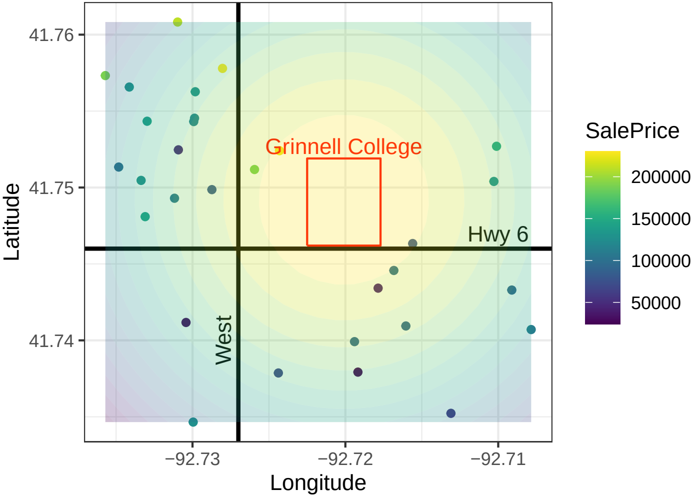
How well would the linear model do if homes closest to the college have the highest price?



Building the KNN "model"

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```
set.seed(10) #Allows for reproducibile random numbers

n <- nrow(GrinnellHouses) #number of observations
prop <- 0.7 #proportion in the training set
train_size <- round(n*prop) #rounded number in training set

train_indices <- sample(1:n, size = train_size, replace = F)
#creates list of indices to include in training

head(train_indices) #an example of a few indices in training set
```

```
## [1] 491 649 330 368 460 439
```

```
GrinnellHouses_train <- GrinnellHouses[train_indices, ]
#subsets for training set by training indices

GrinnellHouses_test <- GrinnellHouses[-train_indices, ]
#subsets all observations not in training set
```

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```r
library(kknn)

House_fit3 <- kknn(SalePrice ~ Longitude + Latitude,
                   train = GrinnellHouses_train,
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House_fit5 <- kknn(SalePrice ~ Longitude + Latitude,
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- Setting `kernel = "rectangular"` corresponds to classic KNN (we'll talk about other options later)

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378	70000	36000	66500.00	63200	71091.9	96097.30	132669.8
686	125000	65500	64666.67	99000	102000.0	91527.30	132674.3
15	58000	124500	131666.67	136900	121356.0	117745.33	132619.4
329	142000	68000	82000.00	85550	79185.0	91638.33	132666.5
23	72000	95000	120000.00	120400	120300.0	118950.00	132653.0
616	189500	285000	198333.33	214000	201200.0	185926.67	133016.0
496	115500	112000	120666.67	117500	135150.0	131875.00	132641.3
869	105000	111000	73000.00	85850	81041.9	97313.97	132666.4
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metric	knn1	knn3	knn5	knn10	knn30	lin_model
RMSE	62850.24	58005.88	57727.92	57129.2	61405.59	79479

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  - There are fewer "similar" houses nearby
  - Not clear how to assess "closeness" if predictors are all on different scales (i.e. *Lat* / *Long* are in angular degrees, but *Year Sold* is in years)

Section 2

KNN Considerations

Quantifying Distance: Euclidean

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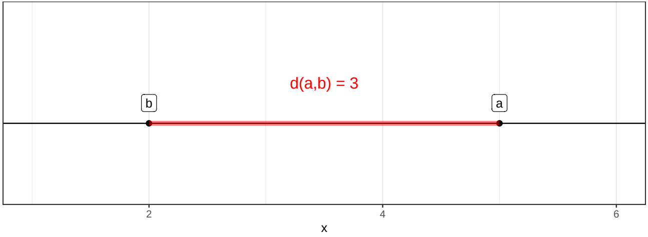
- In 1 dimension (numbers on a line), the Euclidean Distance between two numbers a and b is their absolute value $d(a, b) = |a - b|$



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- In 2 dimensions (points on a plane), the Euclidean Distance between two points $\mathbf{a} = (a_1, a_2)$ and $\mathbf{b} = (b_1, b_2)$ is given by the Pythagorean Theorem:

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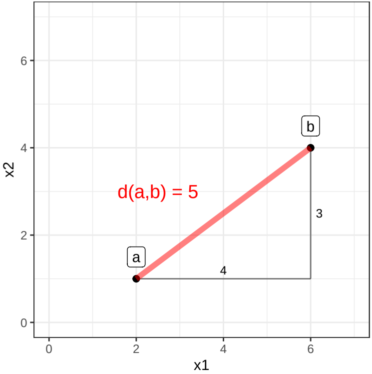

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$$d(\mathbf{a}, \mathbf{b}) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \cdots + (a_p - b_p)^2} = \sqrt{\sum_{i=1}^{p} (a_i - b_i)^2}$$

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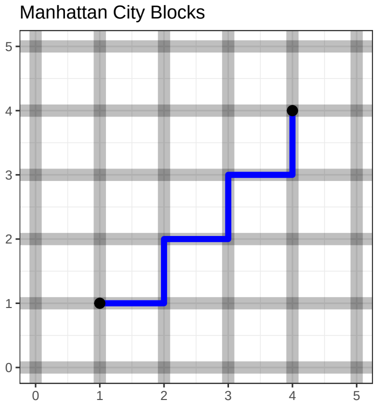

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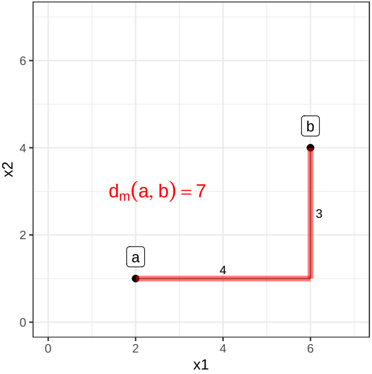

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- The choice of distance function can radically alter predictions.
  - i.e. points that are close in Euclidean distance might not be close in Manhattan distance.

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- In `kknn`, predictors are automatically standardized before predictions are made.

House Prices Redux

Let's use $K = 3, 5, 10, 30$ to make predictions for sale price using year of sale, house age (Old, Mid, Modern), and location.

- We use Manhattan distance, since predictors are incomparable

```
## Example Code
House_fit3_more <- kknn(SalePrice ~ Longitude + Latitude + Age + YearSold,
                   train = GrinnellHouses_train,
                   test = GrinnellHouses_test,
                     k = 3, kernel = "rectangular", distance = 1)


house_lm_more <- lm(SalePrice ~ Longitude + Latitude + Age + YearSold,
                    data = GrinnellHouses_train)

House_fit_lm_more <- predict(house_lm_more, newdata = GrinnellHouses_test)
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170	93000	94000	78666.67	79200	85440	103946.7	93144.97
567	294500	75000	73200.00	97520	127160	142296.7	189978.10
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  - Non-parametric methods tend to perform *worse* that parametric methods when there are small number of observations per predictor in the model